Interacting Particles as neither Bosons nor Fermions: A Microscopic Origin for Fractional Exclusion Statistics

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We study a quantum liquid of particles interacting via a long-ranged two-body potential in three dimensions where the original particles are supposed to be either bosons or fermions. We show that such liquids exhibit the nature of a quantum liquid with fractional exclusion statistics. In both quantum liquids enlarged pseudo-Fermi surfaces are formed from bosons and fermions, although with different excitations. Hence, we conclude that the microscopic origin of exclusion statistics comes from the nature of long-ranged two-body interactions between the particles.

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Bose-Einstein condensation (BEC) \cite{1} and the Fermi-Dirac degeneracy (FDG) \cite{2} in noninteracting quantum gases are the most prominent phenomena in condensed matter systems. In the former, the superfluid transition of the strongly interacting the \textsuperscript{4}He system \cite{3} confirmed our basic understanding of this phase transition since \textsuperscript{3}He system can be regarded as a nonideal Bose gas \cite{4–7} where a delta-function peak exists in the momentum distribution. On the other hand, in the latter, FDG has long been a foundation for solid state physics such as the Sommerfeld theory \cite{8} and Landau’s Fermi liquid theory \cite{9}. This concept has been applied even to the strongly correlated Fermi systems such as high \textit{T_c} superconductivity in the cuprates \cite{10}. However, the superfluid transition and the finite occupation of the zero momentum state—BEC—are distinct phenomena \cite{11} as the strongly correlated Fermi system and the Fermi gas are distinct. To study such differences, experimentalists have recently observed evidences for an ideal BEC \cite{12} (FDG \cite{13}) in a cloud of atomic Bose (Fermi) gases, when a cloud of approximately 200\textsuperscript{87}Rb atoms (one million \textsuperscript{40}K atoms) were cooled to temperatures as low as 20 nK (0.29 \textmu K). Signatures of the BEC (FDG) were seen by relaxing the magnetic trap, allowing the atoms to expand freely, and then imaging the velocity distribution of the atoms in the cloud by laser light.

In spite of the above widely established situations, the role of physical interactions to statistics of the system is unclear. Perturbation methods in the standard arguments \cite{4–7} never provide a change of statistics of the particles. The same is true in the Fermi systems \cite{8,9}. In this Letter, we investigate a microscopic origin for a quantum liquid with fractional exclusion statistics (FES) \cite{14,15} in three dimensions, studying the interacting particles system with long-ranged two-body potential. We show that the interacting bosons or fermions can exhibit the nature of the FES. However, contrary to the expectation that the FES in arbitrary dimensions continuously interpolates between the Bose and Fermi liquids \cite{14–16}, we prove a surprising fact that the FES from the Bose limit is not continuous to that from the Fermi limit in dimensions greater than two except for one dimension \cite{17,18}. The generalization to the other dimensions is straightforward.

Let us first discuss the validity for adopting only the long-ranged two-body potential in the Hamiltonian. Our assumption here is based on the fact that the many-body \textit{S} matrix is essentially determined by the two-body \textit{S} matrix. Without this assumption, generally one cannot in principle determine the statistical properties of a many-body system from only the knowledge of two-body scattering.

We know that this assumption is exactly or approximately true in the following cases: (1) Noninteracting particles; (2) soluble models in one dimension; (3) dilute nonideal gas with virial expansion where usually the two-body interactions are assumed to be short ranged. However, it is well known that for quantum liquids at zero temperature the long-wavelength elementary excitations are exhausted by a single collective mode. Therefore, as far as we are concerned with very long-wavelength physics it is a good approximation to restrict ourselves to the sub-Hilbert space spanned by the collective mode and study the effects of three-body interaction as perturbation—the single mode approximation \cite{19}. Hence, the two-body scattering is good enough to describe the systems in the long-wavelength physics as a starting point. So, we study the following many-body Hamiltonian: \textit{H} = − \textstyle\sum_i \mathbf{\nabla}_i^2 + \textstyle\sum_{i<j} \nu(|\mathbf{r}_i - \mathbf{r}_j|) \mathbf{v}(|\mathbf{r}_i - \mathbf{r}_j|) \text{ the two-body potential between the particles.}

Let us consider the noninteracting systems of bosons and fermions. In this case, following the standard argument \cite{7}, the two-body scattering problem is described by the radial Schrödinger equation for the noninteracting particles is given by [−\textstyle\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2}]u_{kl}(r) = k_n^2 u_{kl}(r), \text{ where } k_n = \sqrt{\frac{\nu}{2\pi}}.\text{ Since the asymptotic form of } u_{kl}(r) \text{ is given by } u_{kl}(r) \sim \text{sin}[kr - \textstyle\frac{l\pi}{2}] \text{ as } r \rightarrow \infty \text{ and the boundary condition } u_{kl}(R) = 0, \text{ the eigenvalues are given by } kr \text{ and } u^{(l)}(0) (u^{(l)} = 0, 1, 2, \ldots), \text{ which provides a spectrum for the noninteracting systems. When there are
$N$ particles in the system, the particles must occupy the states following the statistics of the original particle. This leads one to define the momentum distribution function $g_\alpha(k)$ for Bose ($\alpha = 0$) or Fermi ($\alpha = 1$) gas. Therefore, at zero temperature, we get $g_\alpha(k) = \frac{1}{\pi} \cdot \frac{1}{k^\alpha} \leq k_0^\alpha$. So, the density $d$ and the ground state energy $E_0$ of the system are obtained as $d = \frac{\hbar}{\pi} = \frac{1}{(2\pi)^2} \int |k| \leq k_0 g(k) \cdot d^3k = \frac{(k_0^\alpha)^3}{6\pi^2}$ and $E_0 = \frac{\hbar^2}{2m} \int |k| \leq k_0 k^2g(k) \cdot d^3k = \frac{(k_0^\alpha)^3}{10\pi^2} = \frac{3}{2}(6\pi^2\alpha^2)^{3/2}d^3/3$, respectively, where we have $k_0^\alpha = (6\pi^2\alpha d)^{1/3}$.

Let us now introduce the interactions between the particles. In this case, the radial Schrödinger equation is given by $\left[-\frac{d^2}{dr^2} + V(r) - E_n + \frac{\hbar^2}{m} \frac{d^2}{dr^2}\right]u(k) = 0$. The wave function behaves as $u(k) \cdot \sin(kr - \frac{\hbar}{2m} + \eta(k))$ as $r \to \infty$, where $\eta(k)$ is the phase shift due to the potential. The eigenvalues $k_0^\alpha$'s are determined by the boundary condition such that $kr - \frac{\hbar}{2m} + \eta(k) = \pi n$, where $n = 0, 1, 2, \ldots$ [7]. Following the argument of Sutherland [20], denote the asymptotic momenta by $k$ and denote the noninteracting momenta by $k_0^\alpha = \frac{\hbar}{\pi^3}l$, where $l = (n_x, n_y, n_z)$ is the set of integers, the quantum numbers of the states. The number of orbitals in $d^3k_0^\alpha$ is then given as $\frac{V}{(2\pi)^3}d^3k_0^\alpha$, so that the unperturbed density of orbitals is given by $\frac{1}{(2\pi)^3}$. Then the asymptotic momenta are defined by $k = k_0^\alpha + f(k)$, where $f(k) = -\nabla_k \phi(k)$ and $\phi(k) = \int f_0(\mathbf{k} - \mathbf{k}')d^3k'$. Here we have introduced $f_0(k)$, the density of $k$'s such that $\{k_0^\alpha \}$ in $d^3k_0$ is $V \cdot f_0(k)d^3k$. We now specify the content of the kernel $f_0(k)$. It consists of the two-body phase shift above. Therefore, we have

$$k f_0(k) = -k \phi_0(k) - \sum_k (2l + 1) \eta_l(k). \quad (1)$$

Here Eq. (1) contributes to the induced volume $\Delta$ of the asymptotic momenta due to a single impurity with $\rho(k) = \delta^3(k)\sqrt{\Delta} = 4\pi \rho k f_0(k) = \frac{4\pi}{\hbar} \phi_0(k)$. This corresponds to an apparent increase in the number of states given by $\frac{V}{(2\pi)^3} \Delta = \frac{k^3}{\hbar^3} \phi_0(k)$. If we want pure exclusion statistics, we can identify the quantity as $\gamma = \frac{2}{\pi} \sum_k (2l + 1) \eta_l(\pm \infty)$, which is justified by the Friedel sum rule [21] and the formula of the second virial coefficient [7,22]. Hence, we have $\phi_0(k) = \frac{2\pi^2}{\hbar^2} \frac{\delta f}{\delta k}$ or $f_0(k) = \frac{2\pi^2}{\hbar^2} \frac{\delta f}{\delta k}$. This gives us a three-dimensional electrostatic analogy and by Gauss’s theorem:

$$\int \int_S \mathbf{f}(k) \cdot dS = (2\pi)^3 \gamma \int \int V \rho(k) d^3k. \quad (2)$$

Hence we have $\nabla_k \cdot f(k) = (2\pi)^3 \gamma \rho(k)$ or $\nabla_k \phi(k) = -(2\pi)^3 \gamma \rho(k)$. Using the asymptotic momenta, the density, momentum and energy are given by $\rho = \int \rho(k) d^3k, \quad E = \int k^2 \rho(k) d^3k,$

As a simple example of Gauss’s theorem, let us find the displacement of the Fermi surface. At the Fermi surface, $k_F = k_{0}^\alpha + f(k_F)$, where $k_{0}^\alpha = (6\pi^2\alpha d)^{1/3}$. Applying Gauss’s theorem, $4\pi k_{0}^\alpha f(k_F) = (2\pi)^3 \gamma d$ or $f(k_F) = (2\pi)^3 \gamma k_{0}^\alpha$. The substitution yields $k_{0}^\alpha = k_{0}^\alpha + 2\pi^2 \gamma d$. Then, we can solve for $k_F$ using the roots of a cubic equation as

$$k_F(\alpha) = (6\pi^2\alpha d)^{1/3}, \quad \alpha = \lambda_d, \quad (3)$$

where $\lambda_d = [\alpha^{1/3} + A_+ (\alpha) + A_- (\alpha)]/3$ with $A_+ (\alpha) = (\alpha + \frac{\hbar}{2m} \frac{\hbar^2}{m} \frac{d^2}{dr^2})/3 \gamma d/3$ and $A_- (\alpha) = (\alpha + \frac{\hbar}{2m} \frac{\hbar^2}{m} \frac{d^2}{dr^2})/3 \gamma d/3$. This produces $k_F(0) = (6\pi^2 d/g)^{1/3}$ with $g_B = \gamma/3$ for bosons and $k_F(1) = (6\pi^2 d/g)^{1/3}$ with $g_F = [(1 + A_+ (1) + A_- (1))]/3$ for fermions.

Thus, the interaction between the particles induces the spreading of the Fermi surface. This is due to the fact that the system obeys neither BE statistics nor FD statistics but FES of $g_d$. For the interacting Bose gas, if $\gamma > 0$, then the system acquires the nature of FES of $g_B$. And if $\gamma < 0$, then $k_F$ becomes imaginary, which indicates the system is unstable. On the other hand, for the interacting Fermi gas, if $\gamma > 0(<0)$, then the system acquires the nature of FES of $g_F > 1(<1)$. If we consider the delta-function two-body potential [6], then the phase shift is given by $\eta_0(k) = -ak$ where $a$ is the scattering length that is positive (negative) for a repulsive (attractive) potential. Hence, $\gamma \sim o(a)$.

Let us consider the expansion of the $k$’s for fermions under interaction. A volume $d^3k_0$ containing $\frac{V}{(2\pi)^3}d^3k_0$ orbitals is mapped onto $d^3k$ containing $\frac{V}{(2\pi)^3}d^3k_0$ orbitals. For the ground state, the $k$’s are distributed as densely as possible about the origin, with all orbitals occupied and $|k| < k_F$, therefore these two quantities are the same. Thus, $\rho = \frac{1}{(2\pi)^3} \frac{d^3k}{d^3k_0} = \frac{1}{(2\pi)^3} \frac{d^3k_0}{d^3k} = \frac{1}{(2\pi)^3} \frac{d^3k}{d^3k} \det[\delta_{ij} - \frac{\delta_{ij}}{\delta k_0}] = \frac{1}{(2\pi)^3} \frac{d^3k}{d^3k} \det[\delta_{ij} + \frac{\delta_{ij}}{\delta k_0}]$. Assuming rotational symmetry and introducing $\psi = -\phi/k$, the expression becomes

$$(2\pi)^3 \rho = [(1 - \psi^2)(1 - \psi - k_0 \psi')]. \quad (4)$$

From Gauss’s theorem $\frac{1}{(2\pi)^3} \frac{d}{d^3k} (k^2 \phi') = -(2\pi)^3 \gamma \rho$, we find $k \psi' + 3\psi = (2\pi)^3 \gamma \rho$. Supposing $\psi = \text{const}$ and $\psi' = 0$, we have $\psi = \frac{(2\pi)^3}{3} \gamma \rho$ and hence we get $(2\pi)^3 \rho = (1 - (2\pi)^3/3 \gamma \rho)^3$. Solving for $\rho$ using the roots of a cubic equation, we again obtain $\rho = \frac{1}{(2\pi)^3} \frac{d^3k}{d^3k}$. The corresponding value of $\psi$ is $\psi = \frac{3}{(2\pi)^3}$, while the displacement field $\mathbf{f}(k) = \frac{\gamma}{(2\pi)^3} \mathbf{k} - k_0 \phi \mathbf{k}$. This is due to the fact that the displacement $\mathbf{k} - k_0 \mathbf{k}$ makes sense only for $|k| \leq k_F$, by definition. Thus, $\rho = 0$ for $|k| > k_F$. However, neither $\psi$ nor $\mathbf{f}(k)$ vanishes for $|k| > k_F$. Instead, we find by Gauss’s theorem
\[ \psi_F = \begin{cases} \frac{\gamma}{2\pi^2 k^d} & |k| \leq k_F, \\ \frac{2\pi^2 k^d}{\gamma} & |k| > k_F. \end{cases} \] (5)

Let us next consider the expansion of the \( k \)'s for bosons under interaction. In this case, the definition of Eq. (4) does not work well, since the number of orbitals are located within \(|k| = 0\) at the zero temperature for the ideal Bose gas. Therefore, the volume \( d^3 k_0 \) containing \( \frac{\gamma}{(2\pi)^d} d^3 k_0 \) orbitals is exactly zero in this case. Hence, the definition of \( \rho \) in Eq. (4) apparently provides us zero for \(|k| > 0\). However, Gauss’ theorem should work for this case as well. This gives \( \psi_F = \frac{2\pi^2 k^d}{\gamma} \). To be consistent with the definition, we can invoke a condition \( \psi = 1 \), which yields 
\[ \rho = \frac{1}{(2\pi)^d} \left( |k| \leq k_F \right) \] 
with \( g_B = \frac{\gamma}{2} \). This means that if \( \gamma \to 0 \) then \( \rho \to \infty \), and hence, we recover the ideal Bose gas case. Substituting into \( d = \int \rho(k) d^3k \), we again find the previous result \( k_F = (2\pi^2 \gamma d)^{1/3} \) consistently. We then similarly obtain 
\[ \psi_F = \left\{ \begin{array}{ll} \frac{1}{2\pi^2 k^d} & |k| \leq k_F, \\ \frac{2\pi^2 k^d}{\gamma} & |k| > k_F, \end{array} \right. \] (6)

where the displacement field \( f_B(k) \) is given by \( f_B(k) = \psi_F^k \).

Let us consider excitations near the ground state. In this case, the displacement field \( f(k) \) obtained before plays a crucial role. Suppose that we remove a particle of momentum \( k_B \) with \(|k_B| \leq k_F \) from the ground state and give it momentum \( k_p \) with \(|k_p| > k_F \). Then, all the other momenta are perturbed so that \( k \to k + \frac{1}{\gamma} \delta k \). Substituting this into the definition of the asymptotic momenta, we find 
\[ k + \frac{1}{\gamma} \delta k = k_0 + \int_{|k| \leq k_F} \rho \delta f_0 (k + \frac{1}{\gamma} \delta k - k_0 - \frac{1}{\gamma} \delta k') d^3 k' \] 
Using Taylor expansion, the second term becomes 
\[ \int_{|k| \leq k_F} \rho \delta f_0 (k + \frac{1}{\gamma} \delta k - k_0 - \frac{1}{\gamma} \delta k') d^3 k' \] 
we have used the ground state values of \( \rho \) and \( f_0 \). Expanding all other terms up to \( o(1/\gamma) \), we obtain
\[ \delta f_{tot}(k) = \delta f(k) + f_0 (k - k_p) - f_0 (k - k_B) \] (7)

And \( \delta f(k) \) is given by
\[ \delta f(k) = - \rho \int_{|k| \leq k_F} d^3 k' \times [\delta k' \cdot \nabla k] f_0 (k - k') \] 
\[ = \rho \int_{|k| \leq k_F} d^3 k' \times [\delta k' \cdot \nabla k] f_0 (k - k') \] 
\[ = \rho \int_{|k| \leq k_F} d^3 k' \times [\delta k' \cdot \nabla k] f_0 (k - k') \] 
Using \( \nabla_k \cdot f_0 (k) \) to this, and by Gauss’s theorem, we find
\[ \nabla_k \cdot \delta f_{tot}(k) = - \frac{2\pi^2 \gamma d^3}{3 g_B} \delta^3 (k - k_B) \] 

Let us apply our electrostatic analogy such that the total field \( \delta f_{tot}(k) \) drives the polarization \( \delta \mathbf{k} \) of a linear dielectric through Eq. (7). Applying Gauss’s theorem inside the dielectric, we find 
\[ \nabla_k \cdot \delta f_{tot}(k) = - (2\pi^2 \gamma d^3) (k - k_B) + \rho \nabla_k \delta k \] 
Combining this with Eq. (7), we can write the Poisson equation:
\[ \nabla_k \cdot \delta f_{tot}(k) = - \frac{4\pi q}{\epsilon_a} \delta^3 (k - k_p) \] (8)

inside the dielectrics, where \( q = 2\pi^2 \gamma \) and \( \epsilon_a = 1 + \frac{2\pi \gamma}{\epsilon_r} \). And outside the dielectrics, \( \delta f_{tot}(k) = 4\pi q \delta^3 (k - k_p) \). Thus, the problem is equivalent to the three-dimensional electrostatics problem of a linear dielectric sphere of radius \( k_p \) and dielectric constant \( \epsilon_a \) responding to the field of two point charges: a positive charge \( 2\pi^2 \gamma \) at \( k_p \) outside the sphere and a negative charge \(-2\pi^2 \gamma \) at \( k_B \) inside the sphere. The electric field is represented by \( \delta \mathbf{f}(k) \) while the expression of \( \epsilon_a \) is exactly equivalent to the form of the Clausius-Mossotti equation in classical electrodynamics [23]. By superposition, we can add each of the two contributions, or any other excitations as well, if there are not too many.

We note here a particular feature for bosons. Since \( g_B = \frac{\gamma}{2} \) for bosons, the dielectric constant diverges as \( \epsilon_a \to \infty \). Hence, the pseudo-Fermi sphere of bosons behaves like a perfect conductor, such that the perfect screening of charge occurs inside the sphere. Thus, excitations in the interacting Bose gas can be very different from those in the interacting Fermi gas.

Let us evaluate the shift of momentum and energy:
\[ \Delta P = k_p (-k_B) + \rho \int_{|k| \leq k_F} \delta k \cdot \nabla_k f_0 (k) d^3 k \] 
and 
\[ \Delta E = k_p^2 (-k_B^2) + \rho \int_{|k| \leq k_F} \delta k \cdot \nabla_k f_0 (k) d^3 k \] 
for particles (holes). Using integration by parts and Gauss’s theorem [20], we have 
\[ \Delta P = k_p (-k_B) + \rho \int_{|k| \leq k_F} \delta k \cdot \nabla_k f_0 (k) d^3 k \] 
and 
\[ \Delta E = k_p^2 (-k_B^2) + \rho \int_{|k| \leq k_F} \delta k \cdot \nabla_k f_0 (k) d^3 k \] 
for particles (holes). In the electrostatic analogy, we can interpret \( 2\pi^2 \gamma \rho d k \times \delta k \) as \( k_p^2 \sigma(\Omega) \) as a surface charge density. If \( \theta \) is the angle between the integration variable \( k_F \) and the source of point of either charge \( 2\pi^2 \gamma \) at \( k_p \) or screened charge \(-2\pi^2 \gamma \) at \( k_B \), then we can write as
\[ \Delta P = k_p (-k_B) + \frac{k_p^2}{2\pi^2 \gamma} \int_{|k| \leq k_F} \sigma(\Omega) \cos \theta d\Omega, \] (9)

\[ \Delta E = k_p^2 (-k_B^2) + \frac{2\pi^2 \gamma}{2\pi^2 \gamma} \int_{|k| \leq k_F} \sigma(\Omega) d\Omega, \] (10)

for particles (holes), where \( \Omega = (\theta, \phi) \) and \( d\Omega = \sin \theta d\theta d\phi \). The quantity \( k_p^2 \frac{1}{2} \sigma(\Omega) d\Omega \) is the total surface charge induced on the sphere, while \( k_p^2 \frac{1}{2} \cos \Theta(\Omega) d\Omega \) is the dipole moment of the surface charge induced on the sphere. This leads us to an elementary electrostatics problem. However, in three dimensions, the induced charge problem cannot be solved in a closed form. This is in contrast to the two-dimensional case that can be exactly solvable [20]. Nevertheless, there are two exactly solved limits in the problem: a perfect conductor limit and a plane geometry limit. In our problem here, the former corresponds to bosons, since for bosons the dielectric constant diverges, while the latter corresponds to
When the charge is placed in the medium of outside the sphere, where \( r = \frac{k_p}{2} \). Hence, \( k_F \oint \sigma(\Omega) \times d\Omega = -qr^3 \) and \( k_F \oint \cos \Theta \sigma(\Omega) d\Omega = -ar^2 \). Therefore, we obtain \( \Delta P = k_p - \frac{k_F}{r} \), and \( \Delta E = k_F - \frac{k_F}{r} \).

Eliminating \( k_p \), we find the exact Bogolubov spectrum \([5]\), \( E(p) = pK(p) \) where \( K(p) = \frac{p}{\epsilon} + \left( A + \sqrt{B} \right)^{1/3} + (A - \sqrt{B})^{1/3} \), \( A = \frac{p^2 + k_F^2}{2\epsilon} \), \( B = \frac{p^2 + k_F^2}{2\epsilon} + \frac{1}{4} \), with \( k_F = (2\pi^2 \gamma d)^{2/3} \), while we have no hole excitations. For the fermion case, we can use the method of image charge for a point charge placed in front of the interface between two media of dielectric constants \( \epsilon_1 \) and \( \epsilon_2 \) \([23]\). We then have

\[
\sigma(x) = -\frac{q}{2\pi k_F^2} \frac{\epsilon_2 - \epsilon_1}{e_i (\epsilon_2 + e_i)} \frac{z}{(x^2 + z^2)^{3/2}},
\]

when the charge is placed in the medium of \( \epsilon_1 \) with a distance \( z \) from the surface. Using Eq. (12), we now have \( k_F \oint \sigma(\Omega) d\Omega = \frac{k_F}{r} \oint \cos \Theta \sigma(\Omega) d\Omega = 2\pi k_F^2 \times \int_0^\infty \sigma(x) dx = -\frac{e}{\epsilon} \left( e_i - e_i \right) \) for particles (holes).

Considering the momentum and energy of a pair of particles and holes, we then find \( \Delta P = k_p - k_F \) and \( \Delta E = k_F - \frac{k_F}{r} \) for particles and \( \Delta P = \frac{k_F - k_F}{\epsilon} \) and \( \Delta E = \frac{k_F - k_F}{\epsilon} \) for holes. Hence, we obtain gapless excitations near the pseudo-Fermi surface as \( E(p) = pK(p) + p^2(= 2k_F \cdot p + e_p^2) \) for particles (holes) where we have the same Fermi velocity \( u_F = \frac{h}{\pi k_F} = \frac{h}{m} (6\pi^2 \gamma d)^{2/3} \) for particles and holes.

In conclusion we have investigated the microscopic origin of the FES for the interacting Bose or Fermi liquid in three dimensions. We have applied the electrostatics analogy in the momentum space to reformulate the FES for the interacting systems. We have found the distinct nature of the interacting Bose and Fermi liquid systems although both exhibit the FES due to the long-ranged two-body interaction.

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