Iguchi Replies: In a recent Letter [1] I have formulated the momentum representation for the quantum statistical mechanics of an ideal gas with fractional exclusion statistics in arbitrary dimensions. Using this formalism, I have proved several basic theorems that characterize the nature of this system.

The first two theorems 1 and 2 give some basic facts on the system. Theorem 3 asserts that the grand partition function \( Q \) for an ideal \( g \)-on gas in arbitrary \( D \) dimensions can be factorized as \( Q = Q_F^g Q_B^{1-g} \), and hence the pressure \( P \) becomes additive as \( P = gP_F + (1-g)P_B \). This was used to prove theorem 5 with the aid of the well-known theorem 4, where theorem 5 asserts that there is no condensation of a \( g \)-on gas unless \( g = 0 \) in arbitrary dimensions.

In the preceding Comment, Huang [2] disputes the generality of theorem 3 and has given a proof that theorem 3 is valid only when \( D = 2 \). He presents two approaches to conclude so. The first approach is to use the virial expansion, and the second one is a variational method to consider the additivity of the pressure, which seems to be original.

I agree with his proof that is very clear. Therefore, the factorization of the grand partition function represented in theorem 3 must be a restricted fact to the \textit{two-dimensional} systems as long as we are concerned with such a system of an ideal \( g \)-on gas with a parabolic energy dispersion, \( \epsilon = p^2/2m \), where \( p \) is the \( D \)-dimensional momentum.

However, I would like to make several points: (1) Although theorem 3 is false unless \( D = 2 \), the following theorem 5 in [1] holds valid, since we can prove this by a different method using explicitly the cluster expansion [3]. Therefore, the universality class of the ideal \( g \)-on gases in higher dimensions holds true as well. And the main conclusions in the rest of the Letter remain the same. (2) His second proof relies on only the constant density of states (DOS). For our present case of the ideal \( g \)-on gases with the parabolic dispersion, this constant DOS is realized only when \( D = 2 \). However, there are many systems described by a constant DOS. Therefore, the factorization of the grand partition function can be realized for the class of the systems with a constant DOS, such that the dimension of the system does not matter.

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Comment on “Quantum Statistical Mechanics of an Ideal Gas with Fractional Exclusion Statistics in Arbitrary Dimension”

In a recent Letter on quantum statistical mechanics of an ideal gas with fractional exclusion statistics, Iguchi [1] proved an interesting property that anyons can be regarded as composites of fermions and boson in arbitrary dimension, i.e., in an N-anyon system the thermodynamic potential $Q(a, N)$ can be factorized in terms characteristic of the ideal boson ($a = 0$) and fermion ($a = 1$) gases by the relation $Q(a, N) = (1 - a)Q(0, N_F) + aQ(1, N_f)$, in which $N = N_f + N_b$. In this Comment I first use high-temperature perturbation theory to point out that this relation does not exit in an arbitrary dimension. Next, I prove that in the two-dimensional N-anyon system the thermodynamic potential can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$. Let me now prove that energy $E(a, N)$ is linear in $a$. Using the definition of $W$ we have two useful relations: $d[\ln(1 + W^{-1})]/da = -\beta n$ and $W^{-1}(1 + W)^{-1}dW/da = n[-\beta d\mu/da + \ln(1 + W^{-1})]$. Then, as the density of state $N_d$ is constant in two dimensions we have the relation

$$d[PV\beta]/da = \int d\varepsilon N_d d[\ln(1 + W^{-1})]/da = -\int d\varepsilon N_d W^{-1}(1 + W)^{-1}dW/da$$

$$= -\int d\varepsilon N_d n[-\beta d\mu/da + \ln(1 + W^{-1})]$$

$$= N\beta d\mu/da - kT \int d\varepsilon N_d \ln(1 + W^{-1})/da$$

$$= N\beta d\mu/da + \frac{kT}{2} N_d \ln[1 + W(0)^{-1}]^2.$$

Next, I have evaluated the high-temperature expansion of the energy $E(a, N)$ to order $(\lambda^D)^2$, and the result shows that only in two dimensions can it be linear in $a$. Let me now prove that energy $E(a, N)$ is linear in $a$. Using the definition of $W$ we have two useful relations: $d[\ln(1 + W^{-1})]/da = -\beta n$ and $W^{-1}(1 + W)^{-1}dW/da = n[-\beta d\mu/da + \ln(1 + W^{-1})]$. Then, as the density of state $N_d$ is constant in two dimensions we have the relation

$$d[PV\beta]/da = \int d\varepsilon N_d d[\ln(1 + W^{-1})]/da = -\int d\varepsilon N_d W^{-1}(1 + W)^{-1}dW/da$$

$$= -\int d\varepsilon N_d n[-\beta d\mu/da + \ln(1 + W^{-1})]$$

$$= N\beta d\mu/da - kT \int d\varepsilon N_d \ln(1 + W^{-1})/da$$

$$= N\beta d\mu/da + \frac{kT}{2} N_d \ln[1 + W(0)^{-1}]^2.$$

Now, because $\mu$ is linear in $a$ [2] and $W(0) = e^{-\beta \mu_0}$, we thus see that $d[PV\beta]/da$ does not depend on $a$. This means that the thermodynamic potential $Q(a, N)$ is linear in $a$ and can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$.

In summary, the relation $Q(a, N) = (1 - a)Q(0, N_b) + aQ(1, N_f)$, where $N = N_f + N_b$, does not hold in an arbitrary dimension anyon system, while in a two-dimensional anyon system the thermodynamic potential can be factorized as $Q(a, N) = (1 - a)Q(0, N) + aQ(1, N)$.

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